Ratios and Proportional Relationships and the Number System

As discussed in Module 2, the same six domains appear in multiple grades at the middle school level, as shown in the chart below (which highlights the two domains for this reading).

<table>
<thead>
<tr>
<th>Domain</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratios and Proportional Relationships (RP)</td>
<td></td>
<td>✔️</td>
<td></td>
</tr>
<tr>
<td>The Number System (NS)</td>
<td>✔️</td>
<td>✔️</td>
<td>✔️</td>
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<tr>
<td>Expressions and Equations (EE)</td>
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<tr>
<td>Functions (F)</td>
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<td>Geometry (G)</td>
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<tr>
<td>Statistics and Probability (SP)</td>
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Let’s take a closer look at the Ratios and Proportional Relationships domain and the Number System domain. The table below shows the critical areas and clusters for each grade level (CCSSM, 2010).

<table>
<thead>
<tr>
<th>Ratios and Proportional Relationships (RP)</th>
<th>CCSSM Critical Areas</th>
<th>Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td>Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems</td>
<td>Understand ratio concepts and use ratio reasoning to solve problems.</td>
</tr>
<tr>
<td>Grade 7</td>
<td>Developing understanding of and applying proportional relationships</td>
<td>Analyze proportional relationships and use them to solve real-world and mathematical problems.</td>
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</tbody>
</table>
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The Number System (NS)

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>CCSSM Critical Areas</th>
<th>Clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 6</td>
<td>Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers</td>
<td>• Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</td>
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<tr>
<td></td>
<td></td>
<td>• Compute fluently with multidigit numbers and find common factors and multiples.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Apply and extend previous understandings of numbers to the system of rational numbers.</td>
</tr>
<tr>
<td>Grade 7</td>
<td>Developing understanding of operations with rational numbers and working with expressions and linear equations</td>
<td>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</td>
</tr>
<tr>
<td>Grade 8</td>
<td>[Important but not included as a critical area.]</td>
<td>Know that there are numbers that are not rational, and approximate them by rational numbers.</td>
</tr>
</tbody>
</table>

For more details about the intended outcomes of this domain at each grade level, please visit http://www.corestandards.org/the-standards/mathematics.

Focus and Coherence

Although each state, district, or school may approach the Common Core State Standards for Mathematics (CCSSM) alignment differently, the standards push teachers and their students to understand math as a constantly unfolding story that begins with the earliest concepts of numbers and counting and progresses to the most sophisticated mathematics.

This brings us back to the five building blocks of math: numbers, place value, whole number operations, fractions and decimals, and problem solving. A teacher’s robust understanding of these building blocks and how they inform the coherence and focus of math is paramount, because students will struggle in middle school with the
ratios and proportional relationships and number systems standards if they lack the prerequisite understanding.

To address such problems and provide a rich classroom experience, the Common Core doesn’t require teachers to toss out practices they know to be effective. In fact, the structure of the CCSSM (and the fact that it’s new) may actually free up math teachers to try new techniques.

One question that no math teacher has been spared is, “When am I ever going to use this?” In fact, the question of relevance is at the heart of the Common Core initiative, with a premium placed on the development of mathematical thinking through real-life problem solving. Dan Meyer also illustrates the benefit of real-life problem solving in his TED Talks video.

The use of manipulatives or online simulations can also help make the problems feel more relevant, while helping the students understand the concepts more deeply. Teachers should be sure to follow the guidelines on using manipulatives and online tools provided earlier in this course. While working with manipulatives, students need the guidance of the teacher, who intentionally bridges the concepts with calculations and real life.

**Assessing Understanding**

As teachers incorporate new content into the curriculum and provide a deeper focus on mathematical practice, it is vital to assess what students understand and where they might be facing challenges long before the grading period draws to a close. For this reason, formative assessment should become a central part of mathematics instruction.

A formative assessment is an embedded activity that provides ongoing feedback with the goal to improve instruction and enhance learning. This is different from the more traditional, summative assessment, such as the test at the end of a grading period that is used to determine a grade or to evaluate the outcome of learning.
As Stephen and Jan Chappuis (2008) write, formative assessment “delivers information during the instructional process, before the summative assessment. Both the teacher and the student use formative assessment results to make decisions about what actions to take to promote further learning. It is an ongoing, dynamic process that involves far more than frequent testing, and measurement of student learning is just one of its components.” Formative assessment, for example, can tell a teacher that his student missed the move from negative fractions to rational numbers and help him provide ongoing, targeted support as he works with the student to correct the gap.

Formative assessment also fits well with the Common Core State Standards’ approach to focus and coherence (something that both teacher and student must bring to the classroom), as well as the practice standards. For example, a teacher introducing unit rate (standard 6.RP.3) could first introduce the basic concept and then have students state what basic math skills they think they need to know before they understand how to solve the problem. This approach incorporates a number of practice standards (including 3, 7, and 8); meanwhile, the depth and sophistication of the students’ responses can inform the teacher’s next steps in providing targeted instruction to individual students.

How teachers assess students should also incorporate real-world examples and challenge students to express their understanding and creative problem solving. Although summative assessment will likely remain a part of most classrooms, nontraditional techniques, such as project-based assessments, are key to assessing the actual skill or practice for which instruction was provided. Although the assessments for the CCSSM are still being developed, they will likely focus very heavily on the practice skills rather than on the content skills.

Assessing for Ratios and Proportional Relationships

Here are a few things to look for when assessing for understanding of the ratios and proportional relationships standards:
Can students find equivalent ratios using the 12 x 12 multiplication chart? Do they understand what is happening as they travel across the multiplication chart making equivalent fractions?

Do students apply knowledge of equivalent fractions to finding equivalent ratios?

Do students demonstrate knowledge that ratios can show different relationships between quantities (i.e., part to whole, whole to part, or part to part)?

Do students understand that a proportion is a statement in which two ratios are equal?

Can students find a unit rate (a ratio with a denominator of one) and then use the unit rate to solve a proportion?

Do students understand that a percentage is the rate per 100 units?

Can students calculate the percentage after being given a ratio?

Can students calculate the “part” when given the “whole” and the percentage?

Can students find the “whole” when given a percentage and a part?

Can students apply unit rates to solving pricing, speed, and measurement conversion problems?

Assessing for Number System

Here are a few things to look for when assessing for understanding of the number system standards:

Do students correctly say a decimal numeral?

Are students able to state the correct value of each digit within a decimal fraction?
• Can students build more than one representation of a decimal fraction?
• Do students understand the difference between a factor and a multiple?
• Do students have effective and efficient methods for doing multidigit computations?
• Can students explain why the algorithm for dividing fractions makes sense?
• Do students demonstrate an understanding of negative numbers and can they explain the four operations when working with negative numbers?
• Do students understand absolute value?

Regardless of the concept being assessed, the key is to evaluate the actual skill or practice for which instruction was provided. For example thinking back to the Concrete-Representational-Abstract technique, if students have only been taught how to determine equivalent ratios using pattern blocks (working on the concrete level), then giving them ratios written as fractions on paper and asking them to determine equivalent ratios (practice on the abstract level) is not an appropriate test. Students should emerge from the class with a conceptual and procedural understanding of the content, and any test should assess in kind.

**Errors and Misconceptions**

Let’s start by distinguishing between an error and a misconception. An error is usually a one-off occurrence where the students understand the algorithm but there is a computational error due to carelessness. Misconceptions, on the other hand, are usually observed frequently and consistently. For example, students have misleading ideas or misapply concepts or algorithms.

The most common misconceptions involve incorrectly applying a procedure or an algorithm students’ learned by rote memorization. Such errors occur when students have not developed the mathematical reasoning that accompanies constructing the
mental patterns of concepts; procedures and facts learned only by rote memory do not successfully transfer to new situations (Willis, 2010). To fix a misconception, teachers have to identify the precise source of the issue and address it directly. One way to identify a misconception is to have students use multiple representations for a concept and see if they can demonstrate understanding with all of the representations, some of the representations, or none at all.

To break misconceptions, teachers can use tangible experiences, including manipulatives and other strategies that provide the opportunity to use multiple representations, leading to deeper understanding of mathematical concepts. Teachers can use formative assessment during this process to monitor the students’ learning and ensure that the misconceptions are not embedded further.

Common Ratios and Proportional Relationships Misconceptions

Common misconceptions for this domain include:

**Misconception #1—Ratios:** Students do not realize, for example, that the ratio of 2:8 is the same as the ratio 1:4.

**Misconception #2—Map Scale:** Increasing map scale increases the distance between any two points on the map.

**Misconception #3—Direct Versus Proportional Division:** Mistakes occur when direct instead of proportional division is used. For example, if it takes 2 people 4 hours to do a certain task, students may mistakenly think that it would take 1 person 2 hours rather than 8 hours.

**Misconception #4—Common Additive Misconceptions:** A common misconception in ratio problems is to use addition or subtraction where multiplication or division is appropriate (i.e., using proportional reasoning).

Here’s an example of misconception #4. Given the problem

If 4 U.S. dollars is equal to 3 Canadian dollars, how many Canadian dollars are equal to 20 U.S. dollars?
Some students would say that, because 1 must be added to 3 to get 4 (the difference between Canadian and U.S. dollars), then 20 Canadian dollars must equal 21 U.S. dollars. The student added instead of multiplying.

The correct answer can be calculated by finding the ratio of U.S. dollars to Canadian dollars, 4:3, or 4/3 in this case and then setting up a proportion: 4/3 = 20/x, and finding the equivalent ratio or fraction using multiplication and division. In this case the correct answer is 15 Canadian dollars.

**Common Number System Misconceptions**

Common misconceptions for this domain include:

**Misconception #1—Fractions and Decimal Numerals:** Applying fraction conventions to decimal numerals. For example, the fraction 7 1/4 means 7 + 1/4. The student incorrectly applies this concept when given the number six tenths and writes it as 6.1 or when given the number sixteen tenths and writes it as 16.10. To correct this, review the meaning of fractions and decimal numerals as well as place value.

**Misconception #2—Whole Numbers and Decimal Numerals:** Applying whole number conventions to decimal numerals. Students with this misconception often disregard the decimal and order the decimal numerals by the number of digits rather than the value. They may often align the digits rather than the decimal points when performing subtraction and addition.

To counter this misconception, here are a few things to try:

- Ensure that students say decimal numerals correctly—for example, 6.1 as “six and one tenth.” Saying “six point one” does not reinforce the place value.
- Use a variety of concrete models, such as base 10 blocks, money, and meter sticks, to represent decimal numerals.
• Provide opportunities to recognize that the value of a decimal numeral can be named different ways. For example, 28.1 may represent 28 and one tenth, 281 tenths, or two 10s, eight 1s, and one tenth.

**Misconception #3—The Negative Sign:** Misunderstanding the negative sign and what it applies to, such as thinking that \(-8 + 6 = 2\). In this case, the student made the negative sign into a minus sign, transferring the negative sign from the 8 to the 6.

A related misconception is applying the rules from multiplying and dividing signed numbers. For example, \(-8 + 6 = -14\), which is thinking that a negative and positive always give you a negative.

To correct this type of misconception, try the following:

• Use the number line to show the difference in direction for positive and negative numbers.

• Put a number line on the floor and have the students physically walk the number of steps called out in the correct direction based on the sign of the number called out.

• Provide many models for signed numbers, such as stackable cubes of two different colors, a vertical number line (up is positive, down is negative), a helium balloon that floats or skinks by adding and removing weight, or a picture of a mountain that starts at sea level (zero).