Geometry and Measurement and Data

As the chart below shows, the Geometry (G) and the Measurement and Data (MD) domains appear in all five grades. The highlighted fields indicate the domains we will cover in this module.

Domain	Grade K	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Counting and Cardinality (CC)						
Operations and Algebraic Thinking (OA)						
Numbers and Operations in Base 10 (NBT)						
Number and Operations–Fractions (NF)						
Geometry (G)						
Measurement and Data (MD)						

Let's take a closer look at to the Geometry and Measurement and Data domains. The table below shows the critical areas and clusters for each grade level (CCSSM, 2010).

Geometry (G)		
Grade Level	**CCSSM Critical Areas**	**Cluster**
Grade K	Describing shapes and space	• Identify and describe shapes. • Analyze, compare, create, and compose shapes.
Grade 1	Reasoning about attributes of, and composing and decomposing geometric shapes	Reason with shapes and their attributes.

Grade 2	Describing and analyzing shapes	Reason with shapes and their attributes.
Grade 3	Describing and analyzing two-dimensional shapes	Reason with shapes and their attributes.
Grade 4	Understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.
Grade 5	Developing understanding of volume	• Graph points on the coordinate plane to solve real-world and mathematical problems. • Classify two-dimensional figures into categories based on their properties.
Measurement and Data (MD)		
Grade Level	**CCSSM Critical Areas**	**Clusters**
Grade K	Representing, relating, and operating on whole numbers, initially with sets of objects	• Describe and compare measurable attributes. • Classify objects and count the number of objects in categories.
Grade 1	Developing understanding of linear measurement and measuring lengths as iterating length units	• Measure lengths indirectly and by iterating length units. • Tell and write time. • Represent and interpret data.
Grade 2	Using standard units of measure	• Measure and estimate lengths in standard units. • Relate addition and subtraction to length. • Work with time and money. • Represent and interpret data.

Grade 3	Developing understanding of the structure of rectangular arrays and of area	• Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. • Represent and interpret data. • Geometric measurement: understand concepts of area and relate area to multiplication and to addition. • Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.
Grade 4	{Important but not specifically noted within critical areas at this grade level.}	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles.
Grade 5	Developing understanding of volume	• Convert like measurement units within a given measurement system. • Represent and interpret data. • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

For more details about the intended outcomes of this domain at each grade level, please visit http://www.corestandards.org/the-standards/mathematics.

Geometry

People tend to overlook the value of geometry more than the importance of any other area of mathematics, so teachers are as likely to ask the question as students: With all the focus on numbers and algebra, what happened to geometry?

Geometry is not a mathematical afterthought in the Common Core State Standards. The authors of the standards have taken to heart the ideas stated by J. Michael Shaughnessy (2011), president of the National Council of Teachers of Mathematics:

> If algebra is the language of mathematics, geometry is the glue that connects it… Geometry provides an incredibly rich medium for both mathematical content and mathematical reasoning progressions….Geometry, algebra, and number should be equal, but not separate, because the connections between algebra and geometry are important and powerful for our students…Centuries ago, Descartes provided us with one of the most powerful connections in the history of mathematics when he linked algebra and geometry with his invention of the coordinate plane. Consider that the geometry of graphs of functions brings a visual life to the algebra of their equations. The geometry of functions is easily accessible through the dynamic environments now available to us for teaching with graphing tools in applications on computers and calculators… Missing out on geometry would be a tragedy for many students, since so many blossom in mathematics and enjoy it for the very first time when they encounter geometry. “I finally like mathematics,” they say. “This is really different!”

As Shaughnessy points out, the links between algebra and geometry run deep, with connections including the following:

- The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa.
- The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra.
- Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. (CCSSM, 2010, p. 74)

Assessment of Geometry

The Geometry standards cover a wide range of topics, from shapes to coordinate planes to transformations to lines and angles. Here are a few things to look for when assessing some of the Geometry standards for understanding:

- Are students able to classify shapes in a variety of ways?
- Can students name shapes accurately regardless of their position?
- Can students create regular and irregular polygons?
- Can students relate shapes to real-world objects?
- Can students identify the coordinates for a specific space or point in the coordinate plane?
- When given coordinates, can the student identify the point or space described?
- Can students navigate the coordinate grid by following descriptions for a specific location, distance, and directions? (Bamberger & Schultz-Ferrell, 2010)

Common Geometry Errors and Misconceptions

Because of the wide range of topics covered in geometry, there are quite a few common misconceptions in the geometry domain. Common misconceptions for this domain include the following.

Misconception #1—Shape Orientation: Misclassifying a shape due to its orientation. This often happens when students are only exposed to shapes oriented the same way every time. For example, in math class, how often is a square shown tipped onto one of its corners? If a student usually sees a square sitting on a side, they often will classify a square tipped on its corner as a rhombus.

Some ideas to help prevent or correct this misconception include

- Sending students on a shape hunt.
- Using math-related literature that shows accurate plane figures.
- Having students help develop examples and nonexamples of the basic shapes.
- Taking care that the posters displayed in the classroom show shapes correctly and with their correct names. For example, a rhombus should be labeled a rhombus, not a diamond, and an ellipse should not be labeled as egg-shaped or an oval.
- Asking students to create shapes with a variety of materials to get experience with regular and irregular shapes.
- Incorporate other areas of geometry into the study of shapes, such as tessellations, transformations, and combining shapes. (Bamberger &Oberdorf, 2010)

Misconception #2—Points on a Grid: Incorrectly naming points or spaces on a coordinate grid. Oftentimes, students confuse the X and Y coordinates and count up the *y* axis for the *x* coordinate and vice versa, ending up at the wrong point. In younger students, if they seem to get the closer points incorrect (e.g., graphing (3,4) instead of (4,3)) look to see if the problem is more of a tracking problem than an actual misconception. Have them count to the appropriate number on the *x* axis and then use their finger to move up to the appropriate place described by the *y* coordinate. Sometimes they may have just moved off track. A related misconception is identifying a location on the coordinate grid as the space instead of where there two lines meet.

Some ideas to help prevent or correct this misconception include

- Emphasize that the coordinates are where two lines meet.
- Have the students practice using directional words, such as over, under, left, right, near, and far.
- Create a series of steps using the directional words and have students practice following the directions.
- Set up a scavenger hunt using directional words as clues.
- Tape a grid (coordinate plane) on the floor with all students having their own coordinates. Ask them questions, such as how they'd describe their position in relatation to another student, how far they are from the origin, do they share their coordinates with anyone else, and so on.
- Use activities that involve stories and maps. (Bamberger & Schultz-Ferrell, 2010)

Measurement and Data

Measurement and Data is one domain in which teachers are comfortable using manipulatives or some type of tool, for example, rulers, protractors, coins, or clocks. Bar graphs and other types of graphs are often used as representations of data. Graphs, including pie charts and bar charts, can also be demonstrated with concrete models if students are having difficulty with the concept on paper.

Included in the Measurement and Data domain are the concepts of area and perimeter. These concepts offer great opportunities for the use of concrete models. The area model in particular is crucial to understand, because it is tied to multiplication and other concepts within the CCSMM.

Assessment of Measurement and Data

There are many topics in the Measurement and Data domain. According to Bamberger and Schultz-Ferrell (2010), here are a few things to look for when assessing some of the Measurement and Data standards for understanding:

- Can students correctly place numerals around the clock face?
- Can students count by fives and show understanding of the numerals on the clock and the number of minutes past the hours?
- Can students explain the relationship between the part of the hour that has passed and the numeral on the clock being pointed to?
- Can students correctly match a given time to an analog and digital clock?
- Can students solve elapsed time problems?
- Can students organize sets of coins before determining the value of the coins?
- Can students trade coins, for example, five pennies for a nickel, two nickels for a dime, and so on?
- Do students understand that when units are small, more are needed to measure and when larger units are used, fewer are needed to measure?
- Can students explain what the numbers on a ruler mean?
- Do students know the difference between area and perimeter?
- Do students represent addition and subtraction of time, length, or fractions with the appropriate conversions?
- Can students explain a graph to others?

- Can students explain how a graph might change if there is a change in data?

Common Measurement and Data Errors and Misconceptions

Due to the wide range of topics covered in the Measurement and Data domain, there are many possible misconceptions. Common misconceptions in the Measurement and Data domain include the following.

Misconception #1—Combining Different Units: When adding or subtracting with different units, for example, inches and feet, students regroup with 10s as with whole numbers.

An example of this is if students add two lengths and get 2'17" and then convert it to 3' 7" (rather than 3' 5"). Or if they need to compare two heights, such as 5' 2" and 3' 8", they convert 5'2" to 4'12" (rather than 4'14").

Some activities to help correct this misconception include

- Give students time to explore with the various units prior to problem solving.
- Have students practice recording different measures in different ways, for example, 4 hours is the same as 240 minutes or 14,400 seconds. (Bamberger & Oberdorf, 2010)

Misconception #2—Value of Coins: When determining the value of coins, students count coins as individual objects rather than using the value of the coin. One related misconception is that larger coins are worth more than smaller coins.

Bamberger and Schultz-Ferrell (2010) offer some ideas to reinforce the understanding of the value of coins:

- Use real coins.

- Skip count by 5s, 10s, and 25s using real coins.
- When counting money, start with only two kinds of coins. Start with the coins with the greatest value first, and then pause briefly before going on to the coins with a different value. For example, if there are three dimes and two nickels, have the students count 10, 20, 30; raise a hand indicating that the students should pause; then point to the nickels and have them continue counting, 35, 40.
- Play games where students get pennies based on a number rolled on a die or a number card picked. Once the students have enough pennies, they trade them in to get a nickel or trade in nickels to get a dime.
- Select a target amount and have students come up with all the possible ways to make that amount with quarters, dimes, nickels, and pennies.