Numbers and Operations in Base 10 and Numbers and Operations–Fractions

As the chart below shows, the Numbers & Operations in Base 10 (NBT) domain of the Common Core State Standards for Mathematics (CCSSM) appears in every grade level for elementary school mathematics. In contrast, the Numbers and Operations–Fractions domain begins in grade 3. The highlighted rows indicate the domains we will discuss in this reading.

Domain	Grade K	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5
Counting and Cardinality (CC)						
Operations and Algebraic Thinking (OA)						
Numbers and Operations in Base 10 (NBT)						
Number and Operations–Fractions (NF)						
Measurement and Data (MD)						
Geometry (G)						

Let's take a closer look at the Numbers and Operations in Base 10 and the Numbers and Operations–Fractions domains. The table below shows the critical areas and clusters for each grade level (CCSSM, 2010).

Numbers and Operations in Base 10 (NBT)		
Grade Level	**CCSSM Critical Areas**	**Cluster**
Grade K	Representing, relating, and operating on whole numbers, initially with sets of objects	Work with numbers 11–19 to gain foundations for place value.
Grade 1	Developing understanding of whole number relationships and place value, including grouping in 10s and 1s	• Extend the counting sequence. • Understand place value. • Use place value understanding and properties of operations to add and subtract.
Grade 2	Extending understanding of base 10 notation	• Understand place value. • Use place value understanding and properties of operations to add and subtract.
Grade 3	Developing understanding of multiplication and division and strategies for multiplication and division within 100	• Use place value understanding and properties of operations to perform multidigit arithmetic.
Grade 4	Developing understanding and fluency with multidigit multiplication, and developing understanding of dividing to find quotients involving multidigit dividends	• Generalize place value understanding for multidigit whole numbers. • Use place value understanding and properties of operations to perform multidigit arithmetic.
Grade 5	Extending division to two-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations	• Understand the place value system. • Perform operations with multidigit whole numbers and with decimals to hundredths.

Numbers and Operations–Fractions (NF)		
Grade Level	**CCSSM Critical Areas**	**Cluster**
Grade 3	Developing understanding of fractions, especially unit fractions (fractions with numerator 1)	Develop an understanding of fractions as numbers.
Grade 4	Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers	• Extend understanding of fraction equivalence and ordering. • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. • Understand decimal notation for fractions, and compare decimals and fractions.
Grade 5	Developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions)	• Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Assessing Understanding

Regardless of the concept being assessed, the key is to assess the actual skill or practice for which instruction was provided. Different districts have different policies on assessments and grading, but as much as possible, teach the students so that they have a conceptual and procedural understanding of the standards, and then test the students as they were taught. Make sure to spend time making the bridges necessary from the concrete to the representational and from the representational to the abstract.

Assessing for Numbers and Operations in Base 10

As previously noted, teachers have numerous assessment strategies at their disposal, which the structure and intent of the CCSSM encourages them to explore.

Here are a few things to look for when assessing for understanding of the numbers and operations in base 10 standards:

- Do students understand the value of each digit rather than looking at the digit in isolation?
- Can students compose and decompose numbers?
- Do students see addition and subtraction as inverse operations?
- Can students verbalize decimal numerals correctly?
- Can students state the value of a specific digit within a decimal fraction?

Assessing for Numbers and Operations–Fractions

Here are a few things to look for when assessing for understanding of the Numbers and Operations–Fractions standards:

- Can students fairly share an area in different ways?
- Can students divide a variety of shapes or objects accurately?
- Can students represent fractional parts in nontraditional ways?
- Do students understand that the denominator represents the number of pieces needed to form the whole?
- Do they understand that the numerator stands for a specific number of pieces of the unit?
- Do they understand that when adding or subtracting fractions with like denominators only the numerator changes?

Errors and Misconceptions

As discussed earlier, while an error can be thought of as a one-off occurrence that doesn't necessary reveal a lack of understanding, misconceptions tend to be frequent and consist. Let's look at a few examples.

Common Numbers and Operations in Base 10 Misconceptions

Common misconceptions for this domain include the following.

Misconception #1—Place Value: Not understanding the value that a digit represents when regrouping or renaming in addition and subtraction problems. This misconception often occurs when students are taught algorithms for adding and subtracting multidigit numbers before they understand the concept of place value and what the digits actually represent.

For example, when adding 35 + 26, some students may give the answer 51. They know that 5+6 =11, but since they can only write one digit down for the ones place, they write down only one of the digits and then move on to add 3+2=5. Their final answer then is 51.

Another example of this misconception is seen when a student adding 35 + 26 gives the incorrect answer of 511. They know that 5 + 6 = 11, so they write down 11, and then add 3 + 2 = 5 and record a final answer of 511.

In both of these examples, the student does not understand the concept of place value; that is, that the 3 and 2 represent 30 and 20.

Another example of this misconception is when given a subtraction problem and one of the digits in the bottom number is greater than the digit above it, as shown below.

$$\begin{array}{r} 44 \\ \underline{-\ 26} \end{array}$$

Some students, knowing that the smaller number must be subtracted from the larger number will “flip” 4 – 6 to be 6 – 4 and arrive at the incorrect answer of 22 rather than 18.

Some possible corrections for this misconception include

- Introduce the concept of grouping by having students count by 2s, 5s, 10s, or 100s.
- Use a 100s or 500s chart for addition and subtraction and to look for patterns.
- Use games in which students bundle groups of objects into 10s and then record the number of bundles of 10s, the number of single objects remaining, and the number it represents. Add bundles of 100s for older students.
- Have students represent a 2-digit number in as many possible ways as they can using only 10s and 1s. This can be done by modeling, pictures, or symbols. Use 3-digit numbers and 100s for older students.
- Use partial sums to solve multidigit addition problems.

Misconception #2—Multiplication: Treating each digit individually rather than considering its value when multiplying two-digit factors.

As covered in the previous reading on Operations and Algebraic Thinking, a common mistake in multiplication is to take the numbers at face value and forget about place value. This tends to occur when students are taught an algorithm without understanding the concept behind the algorithm.

For example, a student does the multiplication problem 35 x 23 and gets the incorrect answer of 75.

$$\begin{array}{r} 35 \\ \underline{\times\ 23} \\ 75 \end{array}$$

The student appears to understand that 5 x 3 = 15, so he writes down the 5 and "carries the one." He multiplies 3 x 2 = 6 and adds the one that he carried to get 7. What happened? The student appears to understand regrouping, but he is not realizing that in a multiplication problem, every digit in a factor needs to be multiplied by each digit in the other factor. Because the student doesn't understand the concept, he does not realize that his answer does not make sense.

Some ideas to reinforce the understanding of multiplication include

- In addition to teaching multiplication as repeated addition, use the rectangular array (area) model. This can be connected to repeated addition to show students that they are getting the same answer with another model. See the strategy modeled earlier in this reading.
- Show students how to "expand" the factors of a given problem and use the partial products strategy, which helps show the connection between the distributive property and multiplication and reinforces place value. This strategy can also be show on a rectangular array model, which will provide a visual of what is happening when performing multiplication with double-digit numbers.

$$\begin{aligned} 46 \times 32 &= (40 \times 30) + (40 \times 2) + (6 \times 30) + (6 \times 2) \\ &= 1{,}200 + 80 + 180 + 12 \\ &= 1{,}472 \end{aligned}$$

Misconception #3—Decimals: Applying whole number concepts to decimal numerals.

As students move into studying decimal numerals as a form of fractions, they often try to apply whole number concepts, such as the number with more digits must be larger. This, however, doesn't work with decimals. This leads to difficulties in ordering decimals. Students may also try to align digits, instead of aligning the decimal points, when doing addition and subtraction.

Some ideas to reinforce the understanding of decimals include

- Insist that students read and say decimals correctly. For example, 4.6 should be read as "four and six-tenths," not "four point six."
- Use models to represent decimal fractions. Base 10 blocks, 10x10 grids, money, and meter sticks offer a good variety and show connections between decimals and real life.
- Help students understand what moving the decimal point to the right or left means. For example, given the number 6.5, moving the decimal point one place to the right means that the resulting number is 10 times larger than the original one. Conversely, moving the decimal point one place to the left makes the number one-tenth of the original number.
- Reinforce place value by providing activities that help students name decimal numerals in multiple ways. For example, 52.4 represents five 10s, two 1s, and four tenths OR 52 1s and four tenths OR 524 tenths.

Any activities that helps emphasize that the quantity a number represents is the product of its face value and its place value will help correct the misconceptions related to place value.

Common Numbers and Operations–Fractions Misconceptions

Here are a couple of common misconceptions in the fractions domain.

Misconception #1—Equal Parts: Not understanding that fractions are *equal* parts of a whole. This misconception is displayed when a student says, "I want the bigger half!"

As a result of this misconception, students may try to divide nontraditional shapes into equal parts in the same way that regular shapes, such as circles, squares, and rectangles, are divided. Or they may divide an object or set of objects into equal parts, also known as *equipartitioning*. This is a critical concept in mathematics and relates to not only fractions but also to multiplication, division, and measurement as well.

Some ideas to help correct this misconception include

- Focusing on fractions as part of a whole or parts of a set in the early grades. Other interpretations of fractions as the result of division, as the ratio of two quantities and as operators, will be covered in middle school and later.
- Providing opportunities to talk about fractions and to use concrete models to deepen understanding.
- Varying the shapes that are divided into fractions; that is, use more than the traditional rectangles, squares, and circles/pizzas/pies, when dividing objects into equal parts.
- Expanding beyond halves, thirds, and fourths.
- Using grid paper for students to explore different ways of dividing shapes. The grid paper enables a student to confirm that each of the equal pieces of a whole are the same size (same area) even if the shapes are not consistent.
- Providing opportunities to solve word problems of the area (dividing an area equally) and set (evenly distributing a set of objects) models of fractions. (Bamberger & Oberdorf, 2010)

Misconception #2—Whole Numbers: Applying whole number concepts to fractions.

There are several ways to identify this kind of misconception about fractions. For example, students may compare 1/4 to 1/3 and choose 1/4 as the larger fraction, because four is the bigger number. Students don't yet understand that larger the denominator, the smaller each of the parts of the whole. Or because they learned to add and subtract all of the digits of whole numbers, they try to add and subtract the numerators and the denominators in fractions. Another example of this misconception is when students think that multiplying two fractions together will always give a bigger number (Bamberger & Oberdorf, 2010).

Some ideas to reinforce the understanding of fractions include

- Using the number line to represent fractions. It can be used to show the magnitude of fractions as well as to add and subtract fractions with like denominators.
- Giving students the opportunity to visualize and compare fractions.

A common cause for these misconceptions is teaching procedures before students have a conceptual understanding of fractions. For example, students are recording fractions on paper with no understanding of what numerators and denominators actually stand for.